Intervals, Tuning, and Temperament I

In this series of columns I want to share a few ideas about how to introduce aspects of tuning and temperament to students. In so doing I will unavoidably simplify a very complicated subject. My hope is not to *over*simplify and to simplify in a way that completely avoids inaccuracy.

Most organists do not have to do any tuning as such, or at least can do without tuning if they prefer. However, it is very convenient indeed for any organist to be able to touch up a tuning, or to help out with tuning, or to do a bit of tuning of a chamber organ. Also, of course anyone who plays harpsichord has to expect to do all or most of their own tuning. Beyond that, however, it is very useful and enlightening for any organist to understand the role of tuning, temperament, and the nature of different intervals in the esthetics of organ and harpsichord sound and repertoire, and in the history of that repertoire.

Tuning is one of those areas that many people, including, especially, beginning students, tend to find intimidating. It certainly can be complicated, and can, in particular, involve a lot of math, some of it rather arcane (the 12th root of 2 can be involved, for example, or the ratio between 2^7 and 1.5^{12}). However the concepts are straightforward if not exactly simple. I will start from the very basic here – indeed with the question of what a musical sound is, since everything about tuning arises out of that. I myself, who have tuned constantly for over thirty years, still find it useful to revisit the most basic notions about tuning. I believe that any teacher or student will also find it valuable, even if it involves going over some things that he or she already knows. I believe that if these basics are understood very thoroughly then the details of tuning systems and tuning approaches are fairly easy to understand.

What is a musical sound?

Sound travels in waves, and those waves have peaks and valleys spaced at regular intervals. When the peaks and valleys of a sound wave traveling through the air arrive at a solid material they will tend to make it vibrate. Some materials vibrate inefficiently (a block of granite, for example, or a piece of fabric) some, like an eardrum or the diaphragm of a microphone, vibrate very efficiently indeed. In any case, a sound wave will tend to make a solid vibrate at a speed that corresponds to how often – how *frequently* – the peaks and valleys of that wave arrive at the solid. This is what we call the *frequency* of the sound: a very common-sense term. The wavelength of a sound wave is the distance between two successive peaks. The longer this is, the less frequently those waves will arrive at a given object (say, an eardrum), the more slowly they will make that object vibrate, and the lower the frequency of that sound will be. The shorter the wavelength is, the more frequently the peaks will arrive, the faster the vibrations will be, and the higher the frequency will be. This assumes that these two waves are traveling at the same speed as each other. It is also true that the peaks of any given sound wave will arrive at a given place *more* frequently if that wave happens to be traveling more quickly and *less* frequently if that wave is traveling more slowly. (This is an important point to remember in connection with the practical side or organ tuning, as I will mention later on).

It is the frequency of a sound that humans interpret as *pitch*. A sound wave that makes our eardrums vibrate faster we describe as "higher" in pitch than one that makes our eardrums vibrate more slowly. We do not hear *wavelength* directly: we hear frequency. (This is also an important point for organ tuning). Frequency, being a measurement of how often a particular thing happens, is described in terms of how often that thing (vibration) happens per second. This is, of course, just a convention: it could have been per minute, or per year, or per millisecond.

Sounds that we tend to experience as "music" have wavelengths and frequencies that are consistent and well-organized. Other sounds have frequencies and wavelengths that are in many respects random. This is actually a distinction that – even absent oversimplification – cannot be defined perfectly or in a cut-and-dried manner. It is not just scientific, it is also partly psychological and partly cultural. However, for the (also cultural) purpose of thinking about tuning musical sounds, it is enough to describe those sounds as follows: ***a musical sound is one made up of sound waves with a frequency that remains constant long enough for a human ear to hear it, which may be joined by other sound waves with frequencies that are multiples of the frequency of that first wave.*** A conglomeration of sound waves in which the peaks are spaced irregularly will not be heard as music. To put arbitrary numbers to it, a musical sound might have a wave with a frequency of 220 vibrations per second, joined by waves which cause vibrations of 440, 660, 880, 1100, and 1320 per second. (Vibrations per second or cycles per second are abbreviated Hz). In a musical sound, the lowest (slowest, largest wavelength) part of the sound (220 Hz, above) is called the *fundamental*, and the other components of the sound (440 Hz etc.) are called *overtones* or *upper partial tones – upper partials* for short. A sound consisting of only one frequency with no overtones will be heard as a musical sound, however, this is very rare in non-computer-generated music. Essentially every device for producing music produces overtones, some (oboes, for example) more than others (flutes). (By convention we usually label or describe or discuss a musical sound by referring only to its fundamental, but that never implies that there are no overtones).

There is categorically no such thing as an organ pipe or harpsichord string which produces a fundamental with no upper partials. (Though of course the mix and balance of upper partials can vary infinitely). This fact is crucial in the science and art of tuning, and for the relationship between tuning and esthetic considerations.*

What is an interval?

Any two musical notes form some interval with each other. We are accustomed to identifying intervals by the notes' linear distance from each other in the scale and the terminology for common intervals (second, fifth, etc.) comes from that practice. However, in fact intervals arise out of the *ratio* between the frequencies of the fundamentals of two notes. The number of possible intervals that exist is infinite, since the number of possible frequencies is infinite. However, the common intervals in music are some of those in which the frequency ratios are simple: 1:1, 2:1, 3:2, 4:3 and a few others. And of course these are the intervals that have common names: 1:1 is the unison, 2:1 is the octave, 3:2 is the perfect fifth, 4:3 is the perfect fourth, and so on. To put it another way, if we say that two notes are a perfect fifth apart – as in, say, e above middle c and a below middle c – that means that the frequency of the higher note is in the ratio of 3:2 to the frequency of the lower note, or 1½ times that frequency. (a below middle c is often 220 Hz, so e above middle c should be 330 Hz). If two notes are an octave apart, then the frequency of the higher one is twice the frequency of the lower one, for example middle c at 256 Hz and c above middle c at 512 Hz. The names – both of the notes and of the intervals – are arbitrary conventions, the existence of notes with these ratios a natural fact.

The question of why those particular intervals have been important enough to so many people that they have formed the basis of a whole system of music – indeed many different systems – is a complicated one which probably cannot be answered in full. It seems self-evident to people brought up listening to music based on fifths and thirds, etc., that those intervals "sound good" and that they should form the basis for harmony – itself in turn the basis for music. Explanations for this have been sought in the structure of the universe, in various mathematical models, and through neurological research. However, for the purpose of thinking about how to tune intervals on keyboard instruments the interesting and important thing is that the intervals that we use in music and consider consonant are the intervals that are found in the overtone series described above, and in fact found amongst the lower and more easily audible partial tones. The octave (2:1) is the interval between the first upper partial and the fundamental. The perfect fifth (3:2) is the interval between the second and first upper partials. The perfect fourth (4:3) is the interval between the third and second upper partials. The major third (5:4) is the interval between the fourth and third upper partials. This may in fact explain some of the appeal of those harmonies: in a major triad, all of the notes other than the tonic are found in the overtone series of the tonic. (Of course this is only actually true if you accept the notion that notes an octave apart from one another are "the same" note. This appears to be a universal human perception, and has recently been found to be shared by other primates. Possible neurological sources of this perception have also recently been found). For example, starting with the note c, the first few upper partials give the notes c, g, c again, e, g again. These are the notes of the c major triad.

What does it mean for an interval to be in tune?

If intervals are ratios, then there ought to be a simple definition of what it means for an interval to be in tune: the ratio of frequencies should actually be what the theoretical definition of the interval says it should be. Thus if a given note has a frequency of (for example) 368.5 Hz, then the note a perfect fifth above it should have a frequency of 552.75 Hz. Or if a note has a frequency of 8.02 Hz then the note a major third above it should have a frequency of 10.025 Hz. Also, since these commonly used intervals are related to the overtone series, it makes sense to believe that their being really in tune this way is important: if they are not exactly in tune, then, presumably, they fail to correspond exactly to the overtones. And it may be this correspondence to overtones that gives those intervals their artistic meaning and power.

The very last statement above, however, is speculative and perhaps subjective: a proposed value judgment about the effect of a kind of sound. It is also quite possible that some interested parties – listeners, composers, performers, instrument builders – might happen to prefer the sound of a given interval in a tuning that is *not* theoretically correct. It is very common for instruments on which intonation can be shaded in performance (that is, most non-keyboard instruments, including the voice) indeed to be played with a kind of flexible intonation. Notes are moved a little bit up or down to express or intensify something about the melodic shape or the harmony. This is something that keyboard instruments, with limited exceptions on the clavichord, simply cannot do. However it is an idea that can influence choices that are made in setting a keyboard tuning.

So another definition of what it might mean for an interval to be in tune is this: an interval is in tune if it sounds the way that a listener wants it to sound. Obviously this is almost a parody of a subjective definition, but it also might be the closest to a true one. If the tuning of an interval does indeed fit some theoretical definition but the musician(s) hearing that interval want it to sound a different way then, as a matter of real musical practice, it probably should be that other way (that is, assuming careful and open-minded listening). This notion, and in general the interaction between certain kinds of theory and certain kinds of esthetic preferences, have also been important in the history of keyboard tuning.

What is the problem with Keyboards?

The very premise of the existence of keyboard "tuning and temperament" as a subject is that there are special issues or problems with keyboard instruments from the point of view of tuning. Understanding clearly what these problems are is the prerequisite to understanding keyboard tuning systems themselves, to understanding the role of tuning in the history of keyboard repertoire and, should the occasion arise, to engaging successfully in the act of tuning itself.

The first issue or problem is simply *the fact that keyboard instruments have to be tuned*. That is, prior to playing anything on a keyboard instrument a set of hard and fast choices have to be made about what pitch each note will have. This is another one of those things that is perhaps obvious, but still important to notice. Of course the instrument can be tuned differently for another occasion – more readily with a harpsichord or clavichord than with an organ. But at any moment of playing, each note and each interval is going to be whatever it has been set up to be.

The second problem is an extension of the first, and is the crucial issue in keyboard tuning. The number of keys on the keyboard is simply not enough to represent all of the notes that in theory exist. That is, the notion that, for example, c and b# or g# and a♭ are the same as one another is a fiction or, at the very best, an approximation. This is where the math of the so-called "circle of fifths" comes into play. We are all taught that, if you start at any note – say c – and keep moving up by a fifth, you will come back to the note at which you started: c-g-d-a-e-b-f#-c#-g#/a♭-d#/e♭-a#/b♭-f-c. This circle provides a good working description of the way that we *use* these notes, but it glosses over the fact that if the fifths are pure (theoretically correct) it simply doesn't work: the circle is actually a spiral. Going one way ("up") it looks like this: c-g-d-a-e-b-f#-c#-g#-d#-a#-e#-b#-f𝄪-c𝄪-g𝄪, etc. Going the other way ("down") it looks like this: c-f-b♭-e♭-a♭-d♭-g♭-c♭-f♭-b♭♭-e♭♭-a♭♭-d♭♭-g♭♭, etc. If each fifth is really in the ratio of 3:2 – the frequency of the higher note is 1 ½ times the frequency of the lower note – then none of the enharmonic equivalents work. The b# will simply not be at the same frequency as the c, the g♭♭ will not be the same as the f, and so on.** This in turn means that it is impossible to tune all of the fifths on a keyboard instrument pure: not just difficult but literally impossible.

The third issue or problem of keyboard tuning also arises out of the first and exists in a kind of balance or conflict with the second. On keyboard instruments the tuning of one class of interval determines the tuning both of other intervals and of the scale as a melodic phenomenon. If you tune a keyboard instrument by fifths, then the thirds, sixths, etc. will be generated by those fifths. If you tune the fifths pure, the thirds will come out one way, if you tune the fifths something other than pure (as you must with at least some of them) the thirds will come out some different way. This is an esthetic matter rather than (like the second issue) a practical one.

These three issues have defined and determined the choices made in the realm of keyboard tuning over several centuries. Next month I will discuss what some of those choices have been and how they have arisen.

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*Overtones are also important to the art of registration on the organ, and I discussed them in the April 2008 Diapason column dealing with registration.

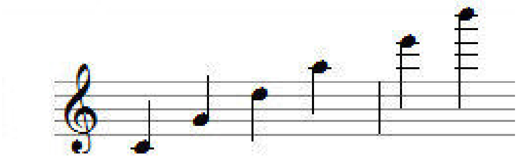
**I have posted a worksheet with some of the actual numbers at the Princeton Early Keyboard Center website: www.pekc.org

Intervals, Tuning, and Temperament II

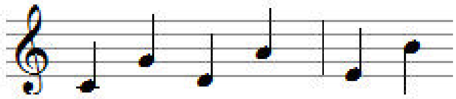
Last month I wrote about some of the fundamentals underlying the art of keyboard temperament: aspects of the nature of musical sound and of intervals, the overtone series, and the so-called circle of fifths. This month I want to discuss keyboard temperament itself using last month's column as a foundation. I will talk about why there has to be temperament, what the major different approaches to temperament have been over the centuries, and some of what the different systems of temperament set out to accomplish. I will talk a little bit about how different temperaments relate to different historical eras, but I will talk more about that next month. Next month I will also talk some about the practicalities of tuning, and about a few miscellaneous matters related to tuning and temperament.

As I said last month, the main point of this brief exposition about tuning and temperament is to help student to become comfortable with the subject and to develop a real if basic understanding of it, regardless of whether they are planning to do any tuning themselves. Before I describe some of the essential details of several tuning systems I want to go through several points about how we discuss tuning and how our thinking about tuning is organized, so that the descriptions of different temperaments will be easy to grasp. These points are as follows:

1) For purposes of talking about tuning, octaves are considered exactly equivalent. (This of course is no surprise, but it is worth mentioning). The practical point of this is that if I say, for example, that "by tuning up by a fifth, six times in a row, I get from c to f#" I do not need to say that I also have to drop the resulting f# down by three octaves to get the simple tritone (rather than the augmented twenty-fifth): that is assumed. To put it another way, simple intervals, say the perfect fifth, and the corresponding compound intervals, say the twelfth or the nineteenth, are treated as being identical to one another.
2) Intervals fall into pairs that are inversions of one another: fifth/fourth; major third/minor sixth; minor third/major sixth; whole tone/minor seventh; semitone/major seventh. For purposes of tuning, the members of these pairs are interchangeable, if we keep direction in mind. For example, tuning up by a fifth is equivalent to tuning down by a fourth. If you are starting at c and want to tune g, it is possible either to tune the g above as a fifth or the g below as a fourth. It is always important to keep track of which of these you are doing or have just done, but they are essentially the same.
3) When, in tuning a keyboard instrument, we tune around the circle of fifths, we do not normally do this:



but rather something like this:



going up by fifths and down by fourths – sometimes up by fourths and down by fifths – in such a way as to tune the middle of the keyboard first, thus creating chords and scales that can be tested.

4) In tuning keyboard instruments we purposely make some intervals impure: that is, not perfectly (theoretically) in tune. When an interval is not pure it is either *narrow* or *wide*. An interval is wide when the ratio between the higher note and the lower note is greater than that ratio would be for the pure interval; it is narrow when the ratio is smaller. For example, the ratio between the notes of a pure perfect fifth is 3:2, that is, the frequency of the higher note is 1 ½ times the frequency of the lower note. In a narrow fifth, that ratio is smaller (perhaps 2.97:2), in a wide fifth it is larger (perhaps 3.05:2). Here's the important point – one that students do not always realize until they have had it pointed out: making an interval *wide* does not necessarily mean making some note *sharp*, and making an interval *narrow* does not necessarily mean making some note *flat*. If you are changing the higher note in an interval, then raising that note will indeed make the interval wider and lowering it will make the interval narrower. However, if you are changing the lower note then raising the note will make the interval narrower and lowering it will make the interval wider.
5) Tuning by fifths (or the equivalent fourths) is the theoretically complete way to conceive of a tuning or temperament system. This is because only fifths and fourths can actually generate all of the notes. That is, if you start from any note and tune around the circle of fifths in either direction, you will only return to your starting note after having passed through all of the other notes. If you start on any given note and go up or down by any other interval, you will get back to your starting note without having passed through all of the other notes.* For example, if you start on c and tune up by major thirds you will return to c having only tuned e and g#/a♭. There is no way, starting on c and tuning by thirds, to tune the notes c#, d, d#, f, f#, g, a, b♭, or b. Tuning is sometimes done by thirds, but only as an adjunct to tuning by fifths and fourths. Any tuning system can be fully described by how it tunes all of the fifths.
6) As I mentioned last month, tuning two or more in a row of any interval spins of at least one other interval. For example, tuning two fifths in a row spins off a whole tone. (Starting at c and tuning c-g and then g-d spins off the interval c-d). Tuning four fifths in a row spins off a major third. (Starting at c and tuning c-g, g-d, d-a, a-e spins off the interval c-e). The tuning of the primary intervals – pure, wide, or narrow – utterly determines the tuning of the resulting (spun-off) interval. For example, tuning four *pure* perfect fifths in a row spins off a major third that is *wider* than the theoretically correct 5:4 ratio: very wide, as a matter of human listening experience. Tuning three *pure* fourths in a row (c-f, f-b♭, b♭-e♭, for example) spins off a minor third that is *narrower* than the theoretically correct 6:5.

So, what is temperament and why does it exist? **Temperament is the making of choices about which intervals on the keyboard to tune pure and which to tune wide or narrow, and about how wide or narrow to make those latter intervals.** Temperament exists, in the first instance, because of the essential problem of keyboard tuning that I mentioned last month: if you start at any given note and tune around the circle of fifths until you arrive back at the starting note, that starting note will be out of tune – sharp, as it happens – if you have tuned all of the fifths pure. The corollary of this is that **in order to tune a keyboard instrument in such a way that the unisons and octave are in tune, it is absolutely necessary to tune one or more fifths narrow.** This is a practical necessity, not an esthetic choice. However decisions about how to address this necessity always involve esthetic choices.

There are practical solutions to this practical problem, and the simplest of them constitutes the most basic temperament. If you start at a note and tune eleven fifths, but do not attempt to tune the twelfth fifth (which would be the out-of-tune version of the starting note) then you have created a working keyboard tuning in which one fifth – the interval between the last note that you explicitly tuned and the starting note – is extremely out of tune. If you start with c and tune g, d, a, etc, until you have tuned f, then the interval between f and c (remember that you started with c and have not changed it) will be a very narrow fifth or very wide fourth. The problem with this very practical tuning is an esthetic, rather than a practical, problem: this fifth is *so* narrow that listeners will not accept it as a valid interval. Then, in turn, there is a practical solution to this esthetic problem: composers simply have to be willing to write music that avoids the use of that interval. This tuning, sometimes called Pythagorean, was certainly used in what we might call the very old days – late middle ages and early Renaissance. As an esthetic matter, it is marked by very wide thirds (called Pythagorean thirds) that are spun off by all of the pure fifths. These thirds, rather than the presence of one unusable fifth, probably constitute the reason that this tuning fell out of favor early in the keyboard era.

The second-easiest way to address the central practical necessity of keyboard tuning is, probably, to divide the unavoidable out-of-tuneness of the fifths between *two* fifths, rather than piling it all onto one of them. For example, if in the example immediately above you tune the last interval, namely b♭-f, somewhat narrow rather than pure, then the resulting final interval of f-c will not be as narrow as it came out above. Perhaps it will be acceptable to listeners, perhaps not. Historical experience has suggested that it is right on the line.

In theory, what I just called the “unavoidable out-of-tuneness” (which is what theorists of tuning call the “Diatonic Comma” or “Pythagorean Comma”) can be divided between or among any number of fifths, from one to all twelve, with the remaining fifths being pure. The fewer fifths are made narrow – that is, “tempered” – the more narrow each of them has to be; the more fifths are left pure (which is the same thing) the easier the tuning is, since tuning pure fifths is the single easiest component of the art of tuning by ear.** The more fifths are tempered, the less far from pure each of them has to be; the fewer fifths are left pure, the more difficult the temperament is to carry out by ear.

Temperaments of this sort, that is, ones in which two or more fifths are made narrow and the remaining fifths are tuned pure, and all intervals and chords are usable, make up the category known as "well-tempered tuning". There exist, in theory, an infinite number of different well-tempered tunings. There are 4083 different possible ways to configure the choice of which fifths to temper, but there are an infinite number of subtly different ways to distribute the amount of out-of-tuneness over any chosen fifths. From the late seventeenth century through the mid to late nineteenth century, the most common tunings were those in which somewhere between four and ten or eleven fifths were tempered, and the rest were left pure. In general, in the earlier part of those years temperaments tended to favor more pure fifths, and later they tended to favor more tempered fifths. The temperament in which all twelve fifths are tempered and the ratio to which they are all tempered is the same (2.9966:2) is known as Equal Temperament. It became increasingly common in the mid to late nineteenth century, and essentially universal for a while in the twentieth century. It was well known as a theoretical concept long before then, but little used, at least in part because it is extremely difficult to tune by ear.

In well-tempered tunings and in fact any tunings, the choices about which fifths to temper affect the nature of the intervals other than fifths. The most important such interval is the major third. The importance of the placement of tempered fifths has always come largely from the effect of that placement on the thirds. Historically, in the period during which well-tempered tuning was the norm, the fifths around c tended to be tempered so as to make the c-e major third close to pure, in any case almost always the most pure major third within the particular tuning. This seems to reflect both a sense that pure major thirds are esthetically desirable or pleasing and a sense that the key of c should be the most pleasing key, or the most restful key, on the keyboard. In general, well-tempered tunings create a keyboard on which different intervals, chords, and harmonies belonging to the same overall class are not in fact *exactly* the same as one another. There might be, for example, major triads in which the third and the fifth are both pure, alongside major triads in which the fifth is pure but the third a little bit wide, or the fifth pure but the third very wide, or the fifth a little bit narrow and the third a little bit wide. It is quite likely that one of the points of well-tempered tuning was to cause any modulation or roaming from one harmonic place to another on the keyboard to effect an actual change in color – that is, in the real ratios of the harmonies – not just a change in the name of the chord or in its perceived distance from the original tonic.

In equal temperament all intervals of a given class are in fact identical to one another, and each instance of a chord of a given type – major triad, minor triad, and so on – is identical to every other instance of that chord except for absolute pitch. Next month I will discuss ways in which the esthetic of each of these kinds of temperament fit in with other aspects of the musical culture of their times.

The other system of tuning that was prevalent for a significant part of the history of keyboard music – from at least the mid sixteenth century through the seventeenth century and, in some places well into the eighteenth – is known nowadays as **meantone tuning**. (This term was not used at the time, and is now applied to a large number of different tunings with similar characteristics). In a meantone tuning, there are usually several major thirds that are unusable wide and one or more fifths that are also unusable. In fact the presence of intervals that must be avoided by composers is greater than in Pythagorean tuning. However, this is in aid of being able to create a large number of pure or nearly pure major thirds. This was, perhaps as a reaction to the earlier Pythagorean tuning with its extremely wide thirds, considered esthetically desirable during this period. The mathematics behind the tuning of thirds tells us that, if two adjacent thirds are both pure, say c-e and e-g#, then the remaining third that is nestled within that octave (see above), in this case a♭-c, will be so wide that no ears will accept it as a valid interval. Therefore only two out of every three major thirds can be pure – that is, eight out of the twelve – and, if they are tuned pure, the remaining major thirds will become unusable. This, of course, in turn means that composers have to be willing to avoid those intervals in writing music. It is striking that composers were willing to do so with remarkable consistency for something like two hundred years.

The distribution of usable and unusable thirds in meantone is flexible. For example, while it is possible to tune c-e and e-g# both pure, as mentioned above, it is also possible to tune c-e and a♭-c pure, leaving e-g# to be unusable. In the late Renaissance and early Baroque keyboard repertoire there are, therefore, pieces that use g# and piece that use a♭, but very few piece that use both. There are pieces that use d# and pieces that use e♭, but very few pieces that use both. There are many pieces that use b♭ and a few that use a#, but almost none that use both. There are very few keyboard pieces from before the very late seventeenth century that do not observe these restrictions. This is powerful evidence that whatever was accomplished esthetically by observing them must have been considered very important indeed.

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*The semitone/major seventh will also generate all of the pitches, but it is essentially impossible to tune by ear and has never been the practical basis of a tuning system.
**more about this next month

Intervals, Tuning, and Temperament III

In the first two columns on tuning I did not refer at all to names of temperaments – neither the rather familiar terms such as "Werkmeister", "Kirnberger", or "Valotti", nor less familiar ones such as "Fogliano-Aron", "Ramos", or "Bendeler". It can be interesting or useful for a student to learn something about these historical temperaments, however there is a reason that I have avoided framing my discussion of temperament with these established tunings. It is much more useful for students to grasp the principals that underlie any keyboard tuning. It is then possible both for the student to understand any specific tuning system – historical or hypothetical – and to invent his or her own, and also to understand some of the practical and artistic implications of different tuning approaches.

Those underlying principals, which I have discussed in the last two columns, can be summarized as follows:

1) It is impossible for all twelve perfect fifths on a normal keyboard instrument to be tuned absolutely pure. This arises out of the mathematics of the fundamental definition of intervals and it is an objective fact. If you start at any note and tune twelve perfect fifths pure, then the note that you come back to – which is supposed to be the same as the starting note – will be significantly sharp compared to the starting note.
2) Therefore, at least one perfect fifth must be tuned narrow. Anywhere from one to all twelve perfect fifths can be tuned narrow, as long as the overall amount of narrowness is correct.
3) The need to narrow one or more fifths is an objective need, and doing so is the *practical* side of keyboard temperament. The choice of which fifths to narrow and (bearing in mind that the overall narrowness must add up to the right amount) how much to narrow them is subjective and is the *esthetic* side of keyboard temperament.

From these principles it is possible to understand, or indeed to re-invent, any of the historical temperaments, each of which is of necessity simply a way of approaching and solving the issues described above. Each of the major historical tuning approaches deals with these issues in a different way:

1) **Pythagorean Tuning.** This is the simplest practical approach, in which eleven fifths in a row are tuned absolutely pure, and the remaining fifth is allowed to be extremely narrow: so narrow that human ears will not accept it as a fifth and it has to be avoided in playing.
2) **Well-tempered Tuning.** In this approach, the narrowness of fifths is spread out over enough fifths that the narrowed fifths sound acceptable to our ears. Practical experience suggests that this means over at least three fifths. The fifths that are not narrowed are left pure. All intervals and thus all chords and all keys are useable.

3) **Meantone Tuning.** In the kind of tuning to which this (modern) term is applied, the tuning of fifth is configured in such a way as to generate pure or relatively pure major thirds. When this kind of tuning was in very widespread use (primarily the 16th and 17th centuries) this was a widely and strongly held esthetic preference. In order to generate a large number of pure major thirds it is necessary to tune a large number of unusable intervals, both thirds and fifths: actually more than in Pythagorean Tuning.
4) **Equal Temperament.** In this temperament, each of the twelve perfect fifths is narrowed by exactly the same amount. In this tuning, alone among all possible keyboard tunings, each specific instance of each type of interval – perfect fifth, major third, and so on – is identical to all other instances of that interval.

All of the above is essentially a summary of some of what I discussed in the last two columns. The rest of this column will be taken up with discussion of some of the artistic and practical implications of different approaches to tuning and temperament, and with brief mention of some interesting historical facts and associations. First, however, I want to talk more about what it means for an interval to be in tune, and specifically about how we listen to an interval to evaluate whether it is in tune.

When two close pitches are sounding at the same time we hear, alongside those notes, a beating or undulating sound that is the difference between the two pitches that are sounding. If a note at 440hz and a note at 442hz are played at the same time, we hear a beating at the speed of twice per second. If the two notes were 263hz and 267hz the beating would be at four times per second. This kind of beating sounds more or less like a (quiet) siren or alarm. It is so much a part of the background of what we hear when we listen to music that most people initially have trouble distinguishing it or hearing it explicitly. Normally once someone first hears beats of this kind, it is then easy be able to hear them and distinguish them thereafter.

These beats are a real acoustic phenomenon. They are not psychological, or part of the physiology of hearing: they are present in the air. If you set up a recording in which one stereo channel is playing one pitch and the other is playing a close but different pitch, then if you play those two channels through speakers into the air, they will produce beats that can be heard. However, if you play them through headphones, so that the two notes never interact with one another in the air but each go directly to a separate ear of the listener, then no beats will be created and the listener will hear the two different pitches without beats.

Notes that are being produced by pipes or strings have overtones. When two such notes are played together, the pitches that mingle in the air include the fundamental and the overtones. Any of those component sounds that are very close to one another will produce beats if they are not in fact identical. It is by listening to these beats and comparing them to a template or plan (either no beats or beats of some particular speed) that we carry out the act of tuning.

For example, if we are tuning a note that is a fifth away from an already-tuned note, then the first upper partial of the higher note is meant to be the same pitch as the second upper partial of the lower note. (For a discussion of overtones see this column from July 2009). If these overtones are in fact identical, then they will not produce any beats; if they are not quite identical they will produce beats. If the goal is to produce a *pure* perfect fifth, then beats should be absent. If the goal is to produce a *narrow* perfect fifth, then beats should be present: faster the narrower a fifth we want. In tuning a major third, the same principal applies, except that it is the third upper partial of the higher note and the fourth upper partial of the lower note that coincide.

Listening for beats produced by coinciding overtones is the essential technique for tuning any keyboard instrument by ear. Any tuning can be fully described by a list of beat speeds for each interval to be tuned. For example, Pythagorean Tuning is a tuning in which the beat speed for each of the eleven fifths that are tuned explicitly is zero. (The twelfth fifth arises automatically). Any Well-tempered tuning can be described by a combination of fifths that have beat speeds of zero and fifths that have various moderate, beat speeds. In Equal Temperament, all the fifths have beat speeds greater than zero, and they all reflect the same ratio, with higher notes having proportionately higher beat speeds. In most Meantone systems, major thirds have no beats or very slow beat speeds, while those fifths that are tuned directly have beat speeds that are similar to those of well-tempered fifths.

These beats have a crucial effect on the esthetic impact of different tuning systems. For example, in Pythagorean Tuning, while all of the perfect fifths are pure (beatless), all of the major thirds are very wide and beat quite fast. This gives those thirds, and any triads, a noisy and restless feeling. A triad with pure fifths and pure thirds – a beatless triad – is a very different phenomenon for a listener, even though it looks exactly the same in music notation. Other sorts of triads are different still: those with a pure major third and a narrow fifth, for example, or with all of the component intervals departing slightly from pure.

General tendencies in the beat structure of different temperaments may explain some things about the history of those temperaments, why they were used at different times, or at least how they correlate with other things that were going on musically at the time when they were current. For example, Pythagorean Tuning was in common use in the late Middle Ages. This was a time when the perfect fifth was still considered a much more consonant or stable interval than the major or minor third. Thus it made sense to use a tuning in which fifths were pure and thirds were wide enough – buzzy enough – to be almost inherently dissonant.

(But it is interesting to speculate about the direction of causality: did Pythagorean organ tuning suggest the avoidance of thirds as consonant intervals, or did a theory-based avoidance of those intervals suggest that a tuning with very wide thirds was acceptable?)

The rise of Meantone Tuning in the late fifteenth century was correlated with the rise of music in which the major third played an increasingly large role as a consonant interval and as a defining interval of both modal and tonal harmony. A major triad with a Pythagorean third does not quite sound like a resting place or point of arrival, but a major triad with a pure third does. During this same period, the harpsichord and virginal also arose, supplementing the clavichord and the organ. These new instruments had a brighter sound with a more explosive attack than earlier instruments. This kind of sound tends to make wide thirds sound very prominent. This may have been a further impetus to the development of new tuning systems in those years.

Meantone tuning, since it includes many unusable intervals, places serious restrictions on composers and players. Modulation within a piece is limited. In general, a given piece can only use one of the two notes represented by a raised (black) key, and must rigorously avoid the other. Many transpositions create impossible tuning problems. Many keys have, as a practical matter, to be avoided altogether in order to avoid tremendous amounts of re-tuning.

Some keyboard instruments built during the meantone era had split sharps for certain notes, that is, two separate keys in, for example, the space between d and e, sharing that space front and back, one of them playing the d# the other playing the e♭. Composers do not seem to have relied on it more than once in a while to write pieces in which they went beyond the harmonic bounds natural to meantone tuning. These split keys were probably intended to reduce or eliminate the need to re-tune between pieces, rather than to expand the harmonic language of the repertoire.

Meantone was no easier to tune than what came before it, or than other tuning systems that were known theoretically at the time but little used, since it placed significant harmonic limitations on composers and improvisers, since it limited transposition, and thus made accompaniment more difficult. Still it remained in use for a very long time. It seems certain that whatever it was accomplishing esthetically must have seemed very important, even crucial. Many listeners even now feel that the sonority of a harpsichord is most beautiful in meantone.

In the late seventeenth century, composers and theorists began to suggest new temperaments that overcame the harmonic restrictions of meantone. These were the well-tempered tunings, in which every fifth and every third is usable as an harmonic interval. In order to achieve this flexibility, these tunings do away with most or, in some cases, all of the pure major thirds. The move to make this change can be seen as a shift from an instrument-centered esthetic – in which the beauty of the sound of the pure thirds was considered more important than perhaps anything else – to a composer-centered esthetic and philosophy, in which limitations on theoretical compositional possibilities were considered less and less acceptable. There were strong defenders of the older tunings well into the eighteenth century. It is interesting that in one well-known dispute about the merits of meantone as opposed to well-tempered tuning, the advocate of the former was an instrument builder (Gottfried Silbermann) and the advocate of the latter was a composer (J. S. Bach).

The crucial esthetic characteristic of well-tempered Tunings is that different keys have different harmonic structures. That is, the placement of relatively pure and relatively impure intervals and triads with respect to the functional harmonies of the key (tonic, dominant, etc.) is different from one key to another. (An interesting experiment about this is possible in modern times. If a piece is recorded on a well-tempered instrument in two rather different keys, say C Major and then E major, and the recordings are adjusted by computer so as to be at the same pitch level as one another, then they will still sound different and be easily distinguishable from each other). There differences are almost certainly the source of ideas about the different inherent characters of different keys. Lists of the supposed emotional or affective characteristics of different keys arose in the very late seventeenth century, at about the same time that well-tempered tuning took hold.

Equal Temperament became common in the mid to late nineteenth century. It is a tuning in which every interval with a given name and every triad or other chord of a particular type is the same as every other interval, triad, or chord with that name or of that type. Part of the appeal of this tuning in the nineteenth century was, probably, its theoretical consistency and symmetry. Many people have found the concept of equal temperament intellectually satisfying: it does not have what might be thought of as arbitrary differences between things that, theoretically at least, ought to be the same. Equal temperament took hold in the same era of organ history that included logarithmic pipe scalings – another theoretically satisfying, mathematically inspired idea. During this same time, designers of wind instruments were working to make those instruments sound the same – or as close as humanly possible – up and down the compass. This is another manifestation of a taste for avoiding seemingly arbitrary or random difference.

On an equal tempered keyboard, the computer experiment described above would result in two indistinguishable performances: it is not possible to tell keys apart except by absolute pitch. The rise and dissemination of equal temperament also coincided with a general world wide increase in travel. In a world in which equal temperament and a particular pitch standard (say a'=440hz) will be found anywhere and everywhere, a flutist, for example, can travel from Europe to America or Japan or anywhere and expect to be able to play with local musicians.

It is also likely that the general acceptance of equal temperament helped lead to twelve-tone and other atonal music by promoting the idea (and the actual listening experience) that all keys and all twelve semitones were the same.

In equal temperament, no interval is pure, and no interval is more than a little bit out of tune. This is a tuning that, just as a matter of taste or habit, appeals strongly to some people and does not appeal to others. I have known musicians with no training (or for that matter interest) in historical temperaments who could not stand to listen to equal temperament because they found equal-tempered thirds grating; I have known others who can accept the intervals of equal temperament as normal but who cannot tolerate the occasional more out of tune intervals of well-tempered tuning.

At the Princeton Early Keyboard Center website there are links to several resources describing and comparing historical temperaments and discussing further some of what I have written about here.

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